

Numerical model for predicting thermodynamic cycle and thermal efficiency of a beta-type Stirling engine with rhombic-drive mechanism

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ABSTRACT

This study is aimed at development of a numerical model for a beta-type Stirling engine with rhombic-drive mechanism. By taking into account the non-isothermal effects, the effectiveness of the regenerative channel, and the thermal resistance of the heating head, the energy equations for the control volumes in the expansion chamber, the compression chamber, and the regenerative channel can be derived and solved. Meanwhile, a fully developed flow velocity profile in the regenerative channel, in terms of the reciprocating velocity of the displacer and the instantaneous pressure difference between the expansion and the compression chambers, is derived for calculation of the mass flow rate through the regenerative channel. In this manner, the internal irreversibility caused by pressure difference in the two chambers and the viscous shear effects due to the motion of the reciprocating displacer on the fluid flow in the regenerative channel gap are included. Periodic variation of pressures, volumes, temperatures, masses, and heat transfers in the expansion and the compression chambers are predicted. A parametric study of the dependence of the power output and thermal efficiency on the geometrical and physical parameters, involving regenerative gap, distance between two gears, offset distance from the crank to the center of gear, and the heat source temperature, has been performed.

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1. Introduction

Stirling engines are power machines that operate over a closed, regenerative thermodynamic cycle, with cyclic compression and expansion of the working fluid at different temperature levels, as described by Walker [1]. The working fluid in the Stirling engines might be air, nitrogen, helium, or hydrogen. In theory, ideal Stirling engines features high thermal efficiency, low emissions, and quietness. Most importantly, a variety of heat sources can be utilized, including solar energy, waste heat, and fossil fuels. In particular, the solar dish power systems (for example, the SES 25-kW SunCatcher [2]) typically consisting a solar dish concentrator, a sun tracker, and a Stirling engine have received great attention from the energy-related researchers in recent years.

In general, the Stirling engines are classified into three types of configuration [3] as follows:

- (1) α -type – this type of configuration features two pistons, each in its own cylinder;
- (2) β -type – this type of configuration has a piston and a displacer in the same cylinder; and

- (3) γ -type – this type of configuration has a piston and a displacer, each in its own cylinder.

In addition, a double-acting Stirling engine has multiple cylinders and elongated power pistons, and can be considered as a coupled α engines with thermodynamic cycle taking place between the top of one piston and the bottom of the next piston.

Stirling engine technology has come a long way in the past several decades. However, new concepts and designs continue to appear, according to the review of Ross [4]. Among all, the domestic-scale Stirling electric power generator [5] is particularly of great market potential.

In an ideal Stirling engine, compression and expansion chambers were assumed to be maintained at constant volume or isothermal conditions during the heating and the cooling processes. However, in 1859 Rankine [6] already mentioned that neither the heating nor the cooling takes place exactly at constant volumes or at constant temperatures. A simple second-order analysis of the ideal Stirling engine cycle was proposed by G. Schmidt in 1871 [7]. The Schmidt analysis has been used widely as an approximation of Stirling engine performance, and Berchowitz and Urieli [8] gave a detailed description of it. The Schmidt analysis takes the isothermal analysis of Stirling engines one step further by assuming that the volume of the expansion and the

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Nomenclature

C_1, C_2	coefficient of velocity equation
C_v	constant volume specific heat (kJ/kg K)
C_p	constant pressure specific heat (kJ/kg K)
d_1	diameter of displacer (m)
d_2	core diameter of cylinder (m)
G	regenerative channel gap (m)
h	enthalpy (kJ/kg)
l_d	height of displacer (m)
l_t	height of the engine (m)
l_1, l_2, l_3, l_4	lengths of the linkages of the rhombic-drive mechanism (m)
L	distance between the two gears (m)
L_{dt}	length from the linkage l_4 to the top surface of displacer (m)
L_{pt}	length from the linkage l_1 to the top surface of piston (m)
m	mass of working fluid (kg)
$\dot{m}$	mass flow rate (kg/s)
P	pressure (kPa)
Q	volumetric flow rate (m ³ /s)
$\dot{Q}_{in}$	input heat transfer rate (kJ/s)
Q_{in}	input heat transfer per cycle (kJ/cycle)
r, φ, z	cylindrical coordinates
r_1	radius of displacer (m)
r_2	core radius of cylinder (m)
R	gas constant (J/kg K)
R_d	offset distance from the crank to the center of gear (m)
R_{t1}	thermal resistance of heating head (°C/kW)

R_{t2}	thermal resistance of cooling jacket (°C/kW)
t	time (s)
t_{max}	maximum computation time (s)
t_p	period (s)
t_0	reference time (s)
T	temperature (K)
T_H	heat source temperature (K)
T_L	heat sink temperature (K)
U	internal energy (kJ/s)
v	velocity (m/s)
V	volume (m ³)
$\dot{W}_{out}$	power output (kJ/s)
W_{out}	network output per cycle (kJ/cycle)
Y	displacement (m)

Subscripts

c	compression chamber
d	displacer
e	expansion chamber
p	piston
r	regenerative channel

Greek symbols

ε	regeneration effectiveness
γ	compression ratio
η	thermal efficiency
μ	dynamic viscosity (kg/m s)
θ	crank angle (rad)
ρ	fluid density (kg/m ³)
ω	rotational speed (rpm)

compression space vary sinusoidally. However, it does not take into account non-isothermal effects and internal irreversibility caused by fluid friction and the pressure difference in different spaces of the engine. As a result, it is not capable of dealing with the thermodynamic phenomena in a real engine.

Recently, the present authors built a 10-W β -type Stirling engine with a rhombic drive (Model 10ST1), of which the photographs are shown in Fig 1. According to the experiences learned from the development of the engine, it is recognized that major fundamental problems that the designers of the Stirling engines have to face were the time consumption in the design process and the cost of fabrication. In most of the cases the determination of the geometrical parameters of the engines is merely based on the experience of the designers or on insufficient costly experimental information. Therefore, the improvement of the engine can only be carried out in a relatively high-cost time-consuming trial-and-error process. In this regard, modeling the engines is expected to be one of the alternative solutions to these issues. A number of numerical models and simulation methods for different types of the Stirling engines have been proposed. The Stirling cycle machines under non-isothermal working condition were firstly analyzed by Finkelstein [9] in 1960s. His analysis was recognized as one significant development in the past several decades. His model, by means of introducing the heat transfer coefficient, the finite heat transfer in the working space was derived. The temperature variation of the gas in the working chambers was investigated. Schulz and Schwendig [10] proposed a general simulation model for Stirling cycles. A thermodynamic analysis of a Stirling engine including dead volumes of hot space, cold space and regenerator was performed by Kongtragool and Wongwises [11]. Most recently, Karabulut, et al. [12] presented a β -type Stirling engine with a displacer

driving mechanism by means of a lever. By means of nodal thermodynamic and kinematic analysis, the instantaneous temperature distribution of working fluid, through the heating–cooling passage, conducting the cold space to hot space, was studied. Parlak, et al. [13] performed a thermodynamic analysis of a γ -type Stirling engine by using a quasi steady flow model. In addition, Mahkamov [14] performed an axisymmetric computational fluid dynamics approach to the analysis of the working process of a solar Stirling engine. de Boer [15] tried to find the aximum attainable performance of Stirling engines and refrigerators. Andersen, et al. [16] presented a control-volume-based modeling in one space dimension of oscillating, compressible flow in the reciprocating machines. Reinalter et al. [17] provided detailed performance analysis of a 10 kW dish/Stirling system.

It has been recognized that these models give a certain amount of information that are not easily obtained by the experiments. In addition, modeling costs much lower since it can be readily used to evaluate the performance of the engine design before the design is submitted for fabrication. The present model is referred to as the second-order model. Herein the three-dimensional CFD model is not adopted because it is an expensive method. Though the three-dimensional CFD model might provide detailed information regarding the flow pattern and temperature and pressure distributions, it requires much more computation efforts than the present model.

The rhombic drive, which is frequently used on a β -type Stirling engine [18], utilizes a jointed rhomboid to convert linear motion of a reciprocating piston to a rotational of fly wheel. In the rhombic drive mechanism, one rigid rod is installed connecting the piston to the top corner of the jointed rhomboid, and another rigid rod connecting the displacer to the bottom corner of the rhomboid. In



Fig. 1. A 10-W β -type Stirling engine with rhombic-drive (Model 10ST1) and its parts [from Power Engines and Clean Energy Laboratory (PEACE Lab.), National Cheng Kung University].

addition, two symmetric gears of equal diameter are connected to the right and the left corners of the rhomboid fixed on the gears at an offset distance from gears center. The two gears are in contact and rotate in opposite directions. When gas pressure is applied to the piston, the top corner of the rhomboid is pushed downward; the rhomboid is flattened in the direction of the piston axis; and then it pushes on the gear wheels and causes them to rotate. At the same time, as the gear wheels rotate, the rhomboid progresses its change of shape and drives its bottom corner and the displacer to move upward.

The purpose of the present study is aimed at development of a numerical model for predicting thermodynamic cycle and efficiency of the β -type Stirling engine with the rhombic drive, in which the annual gap around the displacer acts as a regenerator, so as to evaluate the thermodynamic performance of the engine and predict the periodic variation of pressures, specific volumes, temperatures, masses, and heat transfers in the expansion and the compression chambers. Note that the present model takes into

account the non-isothermal effects, the effectiveness of the regenerative channel, and the thermal resistance in the heating head. Meanwhile, the internal irreversibility caused by pressure difference in the two chambers and the viscous shear effects due to the motion of the reciprocating displacer on the fluid flow in the regenerative channel gap are also included.

2. Numerical model

A schematic diagram of the engine is illustrated in Fig. 2. The working fluid contained in the engine is air. The applied model has been divided into three control volumes: (1) expansion chamber, (2) regenerative channel, and (3) compression chamber. Each of the three control volumes can be treated as an open system. It is assumed that the working fluid is an ideal gas and that the volumes of the chambers as well as the pressures, temperatures, and masses of the air in the expansion and the compression chambers are

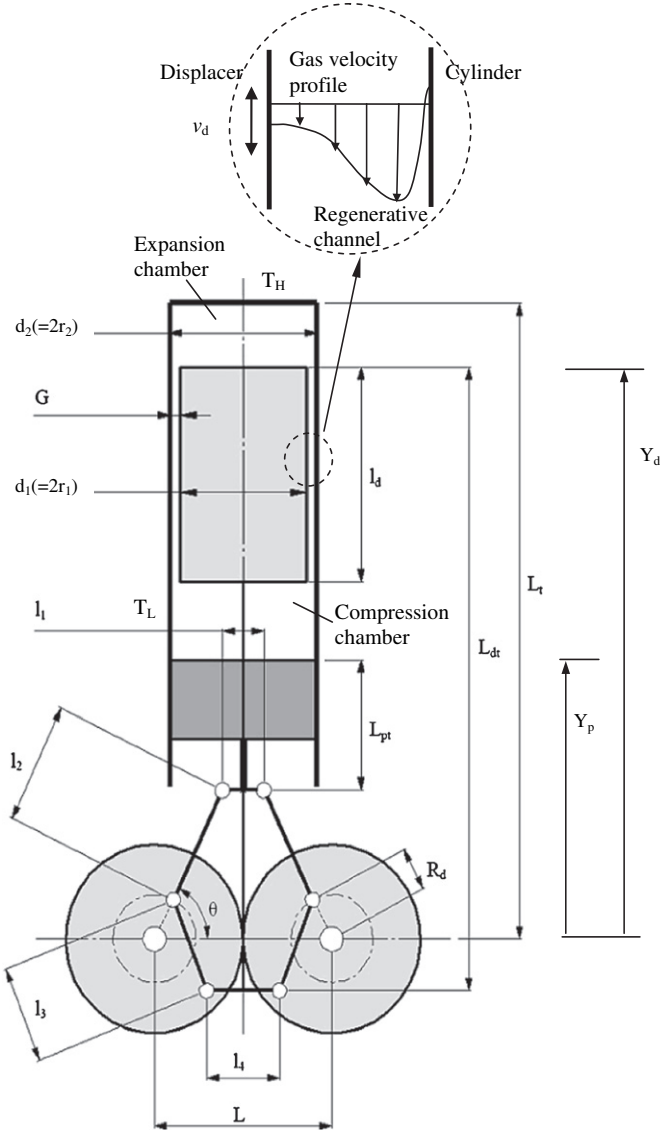


Fig. 2. Schematic of β -type Stirling engine with rhombic drive.

varying with time transiently. However, all the time-varying properties are uniform inside the chambers at any time instant.

Displacements of the piston and displacer connected with the rhombic-drive mechanism, $Y_p(t)$ and $Y_d(t)$, are given as:

$$Y_p(t) = L_{pt} + R_d \sin \theta + \left[l_2^2 - \left(\frac{L}{2} - \frac{l_1}{2} - R_d \cos \theta \right)^2 \right]^{1/2} \quad (1)$$

$$Y_d(t) = L_{dt} + R_d \sin \theta - \left[l_3^2 - \left(\frac{L}{2} - \frac{l_4}{2} - R_d \cos \theta \right)^2 \right]^{1/2} \quad (2)$$

where θ is the crank angle; R_d is the offset distance from gear centers to the joints between the rhomboid and the gears; l_1, l_2, l_3 , and l_4 represent lengths of the linkages of the rhombic-drive mechanism; L_{dt} denotes the length from the linkage l_1 to the top surface of the displacer; and L_{pt} is the length from the linkage l_4 to the top surface of the piston.

The volumes of the expansion and the compression chambers, $V_e(t)$ and $V_c(t)$, can be calculated, respectively, in terms of $Y_p(t)$, $Y_d(t)$, and the cross-sectional area of the cylinder:

$$V_e(t) = \pi \cdot r_2^2 (l_t - Y_p(t)) \quad (3)$$

$$V_c(t) = \pi \cdot r_2^2 (Y_d(t) - l_d - Y_p(t)) \quad (4)$$

where r_2 is the core radius of cylinder. Furthermore, differentiating $Y_p(t)$ and $Y_d(t)$ of Eqs. (1) and (2) with respect to t yields the velocities of the piston and the displacer, $v_p(t)$ and $v_d(t)$, respectively, as

$$v_p(t) = R_d \omega \left\{ \cos \theta - \sin \theta \left[l_2^2 - \left(\frac{L}{2} - \frac{l_1}{2} - R_d \cos \theta \right)^2 \right]^{-1/2} \times \left(\frac{L}{2} - \frac{l_1}{2} - R_d \cos \theta \right) \right\} \quad (5)$$

$$v_d(t) = R_d \omega \left\{ \cos \theta + \sin \theta \left[l_3^2 - \left(\frac{L}{2} - \frac{l_4}{2} - R_d \cos \theta \right)^2 \right]^{-1/2} \times \left(\frac{L}{2} - \frac{l_4}{2} - R_d \cos \theta \right) \right\} \quad (6)$$

2.1. Expansion chamber

The expression of volume of the expansion chamber can be further written by inserting Eqs. (1)–(3) as

$$V_e = \pi r_2^2 \left\{ l_t - \left[L_{dt} + R_d \sin \theta - \left(l_3^2 - \left(\frac{L}{2} - \frac{l_4}{2} - R_d \cos \theta \right)^2 \right)^{1/2} \right] \right\} \quad (7)$$

Using the ideal-gas equation of state, the pressure in the expansion chamber is calculated by

$$P_e = \frac{m_e R T_e}{V_e} \quad (8)$$

where m_e is the mass and T_e is the temperature of the air contained in the expansion chamber, and R is the gas constant.

Next, the air temperature in the expansion chamber (T_e) is calculated based on the energy conservation law:

$$\frac{dU}{dt} \Big|_e = \dot{Q}_{in,e} - \dot{W}_{out,e} - \frac{dm}{dt} \left(h + \frac{v^2}{2} \right) \Big|_e \quad (9)$$

where the values of h and v is chosen in accordance with the direction of the mass flow. Here, the mass flowing out of the expansion chamber is prescribed to be positive. Therefore:

For mass leaving the expansion chamber and entering the regenerative channel:

$$\frac{dm}{dt} > 0, \quad h = h_e, \quad v = v_e \quad (10a)$$

where h_e is the specific enthalpy of the air contained in the expansion chamber, and v_e is the average velocity of the air leaving the expansion chamber calculated with $v_e = |\frac{dm}{dt}| / [\rho_e \pi (r_2^2 - r_1^2)]$.

For mass entering the expansion chamber from the regenerative channel:

$$\frac{dm}{dt} < 0, \quad h = h_j, \quad v = v_j \quad (10b)$$

where h_j is the specific enthalpy of the air exiting the regenerative channel, and v_j is the average velocity of the air calculated with $v_i = |\frac{dm}{dt}|/[\rho_j \pi(r_2^2 - r_1^2)]$. Let

$$\frac{dU}{dt}|_e = \bar{m}_e C_v (T_e^{n+1} - T_e^n) / \Delta t \quad (11)$$

where $\bar{m}_e = (m_e^{n+1} + m_e^n)/2$ and Δt is the time step used in the time marching computation. In this study, Δt is typically assigned to be 1×10^{-6} s.

With the help of Eqs. (10) and (11), Eq. (9) can be discretized into the finite-difference form as

$$T_e^{n+1} = \left(1 - \frac{\Delta t}{\bar{m}_e C_v R_{t1}}\right) T_e^n + \frac{\Delta t}{\bar{m}_e C_v} \left[\frac{T_H}{R_{t1}} - \frac{\bar{P}_e (V_e^{n+1} - V_e^n)}{\Delta t} - \frac{dm}{dt} \left(h_e + \frac{v_e^2}{2} \right) \right], \quad \text{for } \frac{dm}{dt} > 0 \quad (12a)$$

or

$$T_e^{n+1} = \left(1 - \frac{\Delta t}{\bar{m}_e C_v R_{t1}}\right) T_e^n + \frac{\Delta t}{\bar{m}_e C_v} \left[\frac{T_H}{R_{t1}} - \frac{\bar{P}_e (V_e^{n+1} - V_e^n)}{\Delta t} - \frac{dm}{dt} \left(h_j + \frac{v_j^2}{2} \right) \right], \quad \text{for } \frac{dm}{dt} < 0 \quad (12b)$$

where T_H is the heat source temperature; $\bar{P}_e = (P_e^{n+1} + P_e^n)/2$; and the specific enthalpy h is calculated by $h = C_p T$. Note that the input heat transfer rate is defined by $\dot{Q}_{in,e} = (T_H - T_e)/R_{t1}$ and the output power due to volume expansion is determined by $\dot{W}_{out,e} = \bar{P}_e (V_e^{n+1} - V_e^n)/\Delta t$. In addition, R_{t1} is thermal resistance of the heating head, and in this study R_{t1} is set to be $0.5^\circ\text{C}/\text{W}$. However, it is important to mention that the thermal resistance is basically dependent on the heating condition, the materials, and the geometry of the heating head. Hence, for a specific case, further experiments or numerical simulation should be required to determine the value of R_{t1} . Eqs. (12a) and (12b) are employed to calculate the air temperature in the expansion chamber.

2.2. Regenerative channel

In the present model, the gap between the outer surface of the displacer and the inner surface of the cylinder is referred to as the regenerative channel. The model of regenerator used here is a simplified model which assumes that the gas is incompressible throughout the interior chambers. The gas in the regeneration channel enters at one end and exits at another, and the direction and the mass flow rate of the gas flow in the regeneration channel is dependent on the reciprocating velocity of the displacer and the instantaneous pressure difference between the expansion and the compression chambers. Since the regenerative channel is usually rather small, it is reasonable to approximate the fluid flow in the channel as a fully developed flow. It implies that $v_r = v_\phi = 0$, $v_z = v_z(r)$. For a quasi-steady, fully developed, incompressible annular flow, the momentum equation can be simplified as follows.

$$\frac{1}{\mu} \frac{dP}{dz} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) \quad (13)$$

Integrating the above equation yields the velocity inside the channel.

$$v_z = \frac{r^2}{4\mu} \frac{dP}{dz} + C_1 \ln r + C_2 \quad (14)$$

where the coefficients, C_1 and C_2 are determined based on the following boundary conditions:

$$\text{B.C} - 1: \quad r = r_1, \quad v_z = v_d(t) \quad (15a)$$

$$\text{B.C} - 2: \quad r = r_2, \quad v_z = 0 \quad (15b)$$

where the reciprocating velocity of the displacer, $v_d(t)$, is determined by Eq. (6). Then, the velocity profile in the regenerative channel is carried out as

$$v_z = \frac{1}{4\mu} \frac{dP}{dz} \left[r^2 - r_2^2 - \frac{(r_2^2 - r_1^2)}{\ln(r_2/r_1)} \ln \frac{r}{r_2} \right] - \frac{v_d(t)}{\ln(r_2/r_1)} \ln r + \frac{v_d(t)}{\ln(r_2/r_1)} \ln r_2 \quad (16)$$

In the above equation, it is noticed that the effects of pressure difference in the two chambers and the viscous shear effects due to the motion of the reciprocating displacer on the fluid flow in the regenerative channel gap are included. The volumetric flow rate Q can then be calculated as

$$Q = \int_{r_1}^{r_2} v_z (2\pi r) dr = \frac{\pi}{8\mu} \left(-\frac{dP}{dz} \right) \left[r_2^4 - r_1^4 - \frac{(r_2^2 - r_1^2)^2}{\ln(r_2/r_1)} \right] - \frac{2\pi v_d(t)}{\ln(r_2/r_1)} \left[\frac{1}{2} (r_2^2 \ln r_2 - r_1^2 \ln r_1) - \frac{1}{4} (r_2^2 - r_1^2) \right] + 2\pi \frac{v_d(t)}{\ln(r_2/r_1)} \ln r_2 \left[\frac{1}{2} r_2^2 - \frac{1}{2} r_1^2 \right] \quad (17)$$

The mass flow rate through the regenerative channel is

$$\frac{dm}{dt} = \frac{\rho\pi}{8\mu} \left(-\frac{P_e - P_c}{l_d} \right) \left[r_2^4 - r_1^4 - \frac{(r_2^2 - r_1^2)^2}{\ln(r_2/r_1)} \right] - \frac{2\pi v_d(t)\rho}{\ln(r_2/r_1)} \left[\frac{1}{2} (r_2^2 \ln r_2 - r_1^2 \ln r_1) - \frac{1}{4} (r_2^2 - r_1^2) \right] + 2\pi \frac{v_d(t)\rho}{\ln(r_2/r_1)} \ln r_2 \left[\frac{1}{2} r_2^2 - \frac{1}{2} r_1^2 \right] \quad (18)$$

where dP/dz has been approximated with $(P_e - P_c)/l_d$. Using the obtained value of dm/dt , the masses of the air contained in the expansion and the compression chambers can be updated as

$$m_e^{n+1} = m_e^n - \frac{dm}{dt} \Delta t \quad (19a)$$

$$m_c^{n+1} = m_c^n + \frac{dm}{dt} \Delta t \quad (19b)$$

where the superscripts n and $n+1$ represent two consecutive time steps.

The mass conservation principle requires that the mass flow rate through the entire regenerative channel will remain constant. That is,

$$\dot{m}_{in,r} = \dot{m}_{out,r} = \dot{m} = \left| \frac{dm}{dt} \right| \quad (20)$$

Applying energy conservation principle to the regenerative channel, one has

$$-\dot{Q}_{in,r} = \dot{m}(h_{in} - h_{out})|_r - \dot{m} \left(\frac{v_{in}^2}{2} - \frac{v_{out}^2}{2} \right) \Big|_r \quad (21)$$

Due to the upwind effects, the inlet and outlet temperatures of the regenerative channel are different as the air flows in different

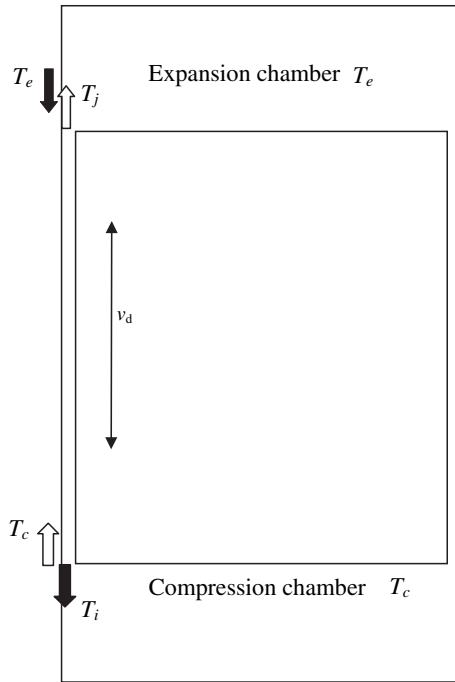


Fig. 3. Inlet and outlet temperatures of air flowing through regenerative channel. (For the air flowing from the expansion to the compression chambers, the inlet temperature is T_e and outlet temperature is T_i . For the air from the compression to the expansion chambers, the inlet temperature is T_c and outlet temperature is T_j .)

directions. Fig. 3 shows the inlet and outlet temperature of air flowing through the regenerative channel. For the air flowing from the expansion to the compression chambers, the channel inlet temperature is T_e and channel outlet temperature is T_i . For the air flowing from the compression to the expansion chambers, the channel inlet temperature is T_c and channel outlet temperature is T_j . Note that the values of the outlet temperatures, T_i and T_j , are strongly dependent on the effectiveness of the regenerative channel.

The regenerator quality is usually defined on an enthalpy basis in terms of a regeneration effectiveness as follows:

$$\varepsilon = \frac{\text{actual enthalpy change through the regenerator}}{\text{maximum theoretical enthalpy change through the regenerator}} \quad (22)$$

Based on the above definition of the regeneration effectiveness, the outlet temperatures, T_i and T_j , are determined by

$$T_i = T_e + \varepsilon(T_c - T_e), \quad \text{for } \frac{dm}{dt} > 0 \quad (23a)$$

and

$$T_j = T_c + \varepsilon(T_e - T_c), \quad \text{for } \frac{dm}{dt} < 0 \quad (23b)$$

respectively. In general, the effectiveness of the regenerator is a function of the geometry and the properties of the porous medium used. The design of the regenerator can be optimized to yield a higher effectiveness [19]. As for the regenerative channel considered here, the regeneration effectiveness is only dependent on the size of the gap (G) and the channel length (l_d). In this study, the channel length l_d is fixed at 0.07946 m. Hence, ε can be assumed

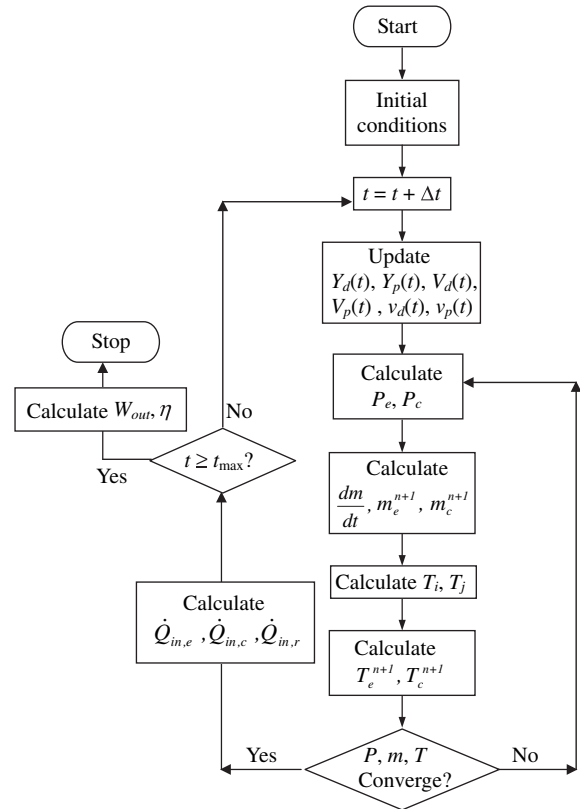


Fig. 4. Flow chart of the computation process.

to be a function of G only. Based on the theory given by Organ [20], the function of ε is approximated as a linear function in terms of G which is varied from 0.1 (at $G = 0.0008$ m) to 0.8 (at $G = 0.0002$ m). Hence, at $G = 0.0005$ m, ε is calculated to be 0.45. The regeneration effectiveness is relatively low since the regenerator here is simply an annular channel. However, note that for a more complicated regenerator, a study of the effectiveness of the regenerator is definitely necessary. In consequence, one has:

For mass flowing from the expansion to the compression chambers:

$$\begin{aligned} \frac{dm}{dt} > 0, \quad T_{in} = T_e, \quad h_{in} = h_e, \quad v_{in} = v_e; \quad T_{out} = T_i, \\ h_{out} = h_i, \quad v_{out} = v_i \end{aligned} \quad (24a)$$

For mass flowing from the compression to the expansion chambers,:

$$\begin{aligned} \frac{dm}{dt} < 0, \quad T_{in} = T_c, \quad h_{in} = h_c, \quad v_{in} = v_c; \quad T_{out} = T_j, \\ h_{out} = h_j, \quad v_{out} = v_j \end{aligned} \quad (24b)$$

In the above equations, as already stated earlier, all the specific enthalpies are calculated by $h = C_p T$ based on the associated temperatures, and all the average velocities of the air are calculated by $v = |\frac{dm}{dt}| / [\rho \pi (r_2^2 - r_1^2)]$.

Eq. (21) is used to determine the heat transfer rate ($\dot{Q}_{in,r}$) between the air flow and the walls of the regenerative channel. As $\dot{Q}_{in,r} > 0$, it means that heat is transferred from the regenerative channel walls to the air, and on the contrary, as $\dot{Q}_{in,r} < 0$, it means that heat is rejected from the air to the channel walls.

2.3. Compression chamber

Introducing Eqs. (1) and (2) to Eq. (4) leads to the volume of the compression chamber:

$$V_c = \pi r_2^2 \left\{ \left[L_{dt} + R_d \sin \theta - \left(l_3^2 - \left(\frac{L}{2} - \frac{l_4}{2} - R_d \cos \theta \right)^2 \right)^{1/2} \right] - \left[L_{pt} + R_d \sin \theta + \left(l_2^2 - \left(\frac{L}{2} - \frac{l_1}{2} - R_d \cos \theta \right)^2 \right)^{1/2} \right] - l_d \right\} \quad (25)$$

From the ideal-gas equation of state, the pressure in the compression chamber is

$$P_c = \frac{m_c R T_c}{V_c} \quad (26)$$

and, the gas temperature in the compression chamber (T_c) is calculated from energy equation

$$\frac{dU}{dt} \Big|_c = \dot{Q}_{in,c} - \dot{W}_{out,c} + \frac{dm}{dt} \left(h + \frac{v^2}{2} \right) \Big|_c \quad (27)$$

where the values of h and v is chosen in accordance with the direction of the mass flow. Please be reminded that the mass flowing into the compression chamber is prescribed to be positive. Therefore:

For mass entering the compression chamber from the regenerative channel:

$$\frac{dm}{dt} > 0, \quad h = h_i, \quad v = v_i \quad (28a)$$

where h_i is the specific enthalpy of the air entering the compression chamber from the regenerative channel, and v_i is the average velocity calculated with $v_i = |\frac{dm}{dt}| / [\rho_j \pi (r_2^2 - r_1^2)]$.

For mass leaving the expansion chamber and entering the regenerative channel:

$$\frac{dm}{dt} < 0, \quad h = h_c, \quad v = v_c \quad (28b)$$

where h_c is the specific enthalpy of the air contained in the compression chamber, and v_c is the average velocity of the air calculated with $v_c = |\frac{dm}{dt}| / [\rho_c \pi (r_2^2 - r_1^2)]$. Again, the time derivative term on the right-hand side of equation is approximated by

Table 1
Geometrical variables of the base-line case (Model 10ST1).

r_2 (m)	0.0205
r_1 (m)	0.02
G (m)	0.0005
$l_1 (= l_2 = l_3 = l_4)$ (m)	0.018
L (m)	0.042
R_d (m)	0.0036
L_{pt} (m)	0.05093
L_{dt} (m)	0.16374
l_d (m)	0.07946
h_i (m)	0.158
Piston stroke (m)	0.01

Table 2
Operating variables of the base-line case (model 10ST1).

Engine speed, ω	2000 rpm
Compression ratio	1.7420
Heat source temperature, T_H	800 K
Heat sink temperature, T_L	300 K
Initial pressure, P_H, P_L	101.3 kPa
Thermal resistance, R_{t1}	0.5 °C/W
Thermal resistance, R_{t2}	0.4 °C/W
Regenerative effectiveness, ε	0.45

$$\frac{dU}{dt} \Big|_c = \bar{m}_c C_v (T_c^{n+1} - T_c^n) / \Delta t \quad (29)$$

where $\bar{m}_c = (m_c^{n+1} + m_c^n)/2$. Note that the input heat transfer rate is defined by $\dot{Q}_{in,c} = (T_L - T_c)/R_{t2}$. Since $T_L < T_c$ usually, the value of $\dot{Q}_{in,c}$ is negative in most of the time. The output power is determined by $\dot{W}_{out,c} = (\bar{P}_c (V_c^{n+1} - V_c^n)) / \Delta t$. Thus, Eq. (27) reduces to

$$T_c^{n+1} = \left(1 - \frac{\Delta t}{\bar{m}_c C_v R_{t2}} \right) T_c^n + \frac{\Delta t}{\bar{m}_c C_v} \left[\frac{T_L}{R_{t2}} - \frac{\bar{P}_c (V_c^{n+1} - V_c^n)}{\Delta t} + \frac{dm}{dt} \left(h_i + \frac{v_i^2}{2} \right) \right], \quad \text{for } \frac{dm}{dt} > 0 \quad (30a)$$

or

$$T_c^{n+1} = \left(1 - \frac{\Delta t}{\bar{m}_c C_v R_{t2}} \right) T_c^n + \frac{\Delta t}{\bar{m}_c C_v} \left[\frac{T_L}{R_{t2}} - \frac{\bar{P}_c (V_c^{n+1} - V_c^n)}{\Delta t} + \frac{dm}{dt} \left(h_c + \frac{v_c^2}{2} \right) \right], \quad \text{for } \frac{dm}{dt} < 0 \quad (30b)$$

where T_L is the heat sink temperature; $\bar{m}_c = (m_c^{n+1} + m_c^n)/2$ and $\bar{P}_c = (P_c^{n+1} + P_c^n)/2$; and R_{t2} is thermal resistance of the cooling jacket, which is assigned to be 0.4 °C/W. Eqs. (30a) and (30b) are used to calculate the air temperature in the compression chamber.

2.4. Network output and thermal efficiency

The network output per cycle is further determined by integrating P with respect to V for a cycle in accordance with the P – V relations taken in the expansion and the compression chambers. That is,

$$W_{out} = \int_{t_0}^{t_0+t_p} P_e dV_e + \int_{t_0}^{t_0+t_p} P_c dV_c \quad (31)$$

The net heat transfer into the air per cycle is a summation of the heat transfers in the expansion chamber, the compression chamber, and the regenerative channel, which are obtained by integrating the heat transfer rates with respect to time for a cycle:

Table 3
Computation results for the base-line case (Model 10ST1).

Work output in expansion chamber (kJ/cycle)	1.236×10^{-3}
Work input in compression chamber (kJ/cycle)	-7.335×10^{-4}
Heat transfer in expansion chamber (kJ/cycle)	3.826×10^{-3}
Heat transfer in compression chamber (kJ/cycle)	-3.288×10^{-3}
Heat transfer in regenerative channel (kJ/cycle)	-3.623×10^{-5}
Net heat input Q_{in} (kJ/cycle)	5.025×10^{-4}
Network output W_{out} (kJ/cycle)	5.025×10^{-4}
Thermal efficiency	0.131
Power output (W)	16.75

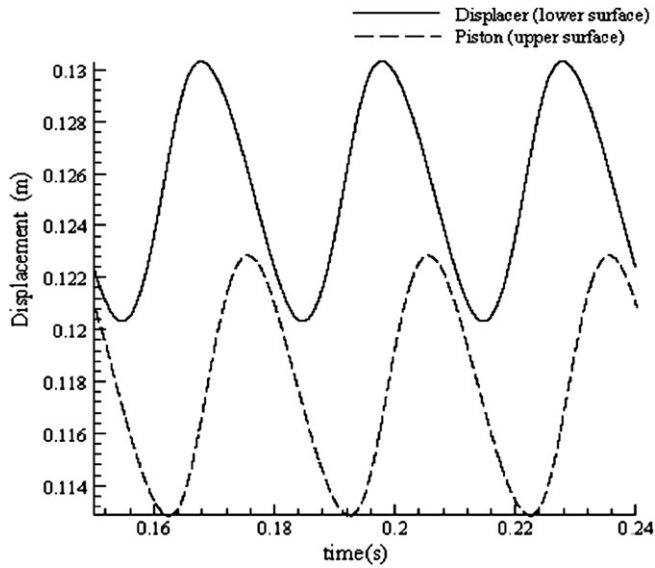


Fig. 5. Displacement of the piston and the displacer.

$$Q_{in} = \int_{t_0}^{t_0+t_p} \dot{Q}_{in,e} dt + \int_{t_0}^{t_0+t_p} \dot{Q}_{in,c} dt + \int_{t_0}^{t_0+t_p} \dot{Q}_{in,r} dt \quad (32)$$

where t_0 is an arbitrarily-selected reference time and t_p represents the period of a cycle. Note that the energy conservation law requires that during the cyclic operation the network output per cycle should be equal to the net heat transfer into the air per cycle. This will be observed from the results later.

The thermal efficiency of the Stirling engine can be expressed as

$$\eta = \frac{W_{out}}{\int_{t_0}^{t_0+t_p} \dot{Q}_{in,e} dt} \quad (33)$$

Fig. 4 shows the flow chart of the computation process. The computation starts from the initial conditions for the air inside the engine. In the beginning, the pressure in the entire engine is set to be 101.3 kPa and the fluid temperature is 300 K. The computation then marches to the next time step and the solutions for pressures, masses, and temperatures in the expansion chamber, the compression chamber, and the regenerative channel are carried out by iteration at each time step. When the time exceeds the required maximum time, the solution process is terminated. The state of the system starts from the initial conditions and then achieves a stable periodic regime eventually. The periodic variation of the properties and heat transfers in different spaces of the engine can then be recorded and the network output and the thermal efficiency are further investigated.

The geometrical and the operating variables of Model 10ST1, which is regarded as the base-line case in this study, are listed in Tables 1 and 2, respectively. In this case, the heat source and the heat sink temperatures are maintained at 800 and 300 K, respectively. The rotational speed of the engine is set at 2000 rpm, and the engine is operated at around atmospheric pressure. Based on the geometrical variables given, the compression ratio is 1.742. Note that some of the variables may be changed to investigate their effects on the performance of the engine while other variables are fixed.

3. Results and discussion

Table 3 shows the computation results for the base-line case. In this table, it is found that the network output per cycle (5.025×10^{-4} kJ/cycle) closely agrees with the net heat transfer into the air per cycle. The numerical results completely comply with the energy conservation law. Furthermore, it is noticed that the engine power output is 16.75 W, and the thermal efficiency reaches merely 13.1%. Such a poor thermal efficiency is frequently encountered with the small-scale engines similar to Model 10ST1. However, the magnitudes of the engine power output and the thermal efficiency may be elevated by modifying the geometrical design or by increasing the regeneration effectiveness of the regenerative channel.

Detailed results from the present analysis of the base-line case are provided in Figs. 5–12. Fig. 5 illustrates the periodic displacement of the displacer and the piston in the cylinder for the base-line case. During the cycles, the displacer moves at a phase angle ahead of the piston in order to draw the working gas traversing back-and-forth between the expansion and the compression chambers for heating or cooling. At some instants, the displacer and the piston were rather close to each other. The minimum distance between the piston and the displacer is treated to be an influential factor affecting the dead zone volume of the engine. Less minimum distance means smaller dead zone in the compression chamber and more working gas in the compression chamber being drawn into the expansion chamber for heating provided the piston and the displacer do not coincide with each other.

Fig. 6 illustrates the volume variation of the expansion chamber and compression chamber. For the base-line case, the volume variation of the compression chamber is larger than that of the expansion chamber. The volume of expansion chamber is varied from 3 to 16.1 cm^3 , whereas the volume of compression chamber is from 2 to 20.3 cm^3 . In the beginning of the plot, the piston and the displacer are moving downwards, and the volumes of both chambers are increased. After a short while, the expansion chamber volume soon reaches its maximum and starts to decrease while the compression chamber volume is increased. When the volume of the expansion chamber reaches its minimum, the volume of the compression chamber reaches its maximum almost at the same time.

Fig. 7 shows the pressure variation inside the expansion and compression chambers. The initial pressure of the two chambers is

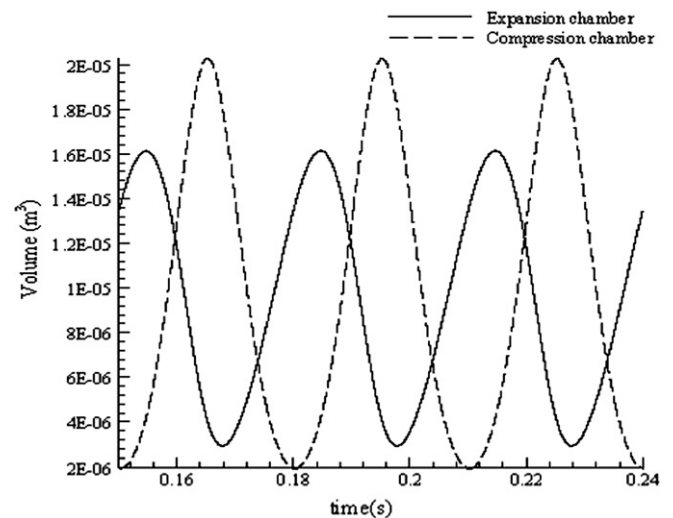


Fig. 6. Volume variation of the expansion and the compression chambers.

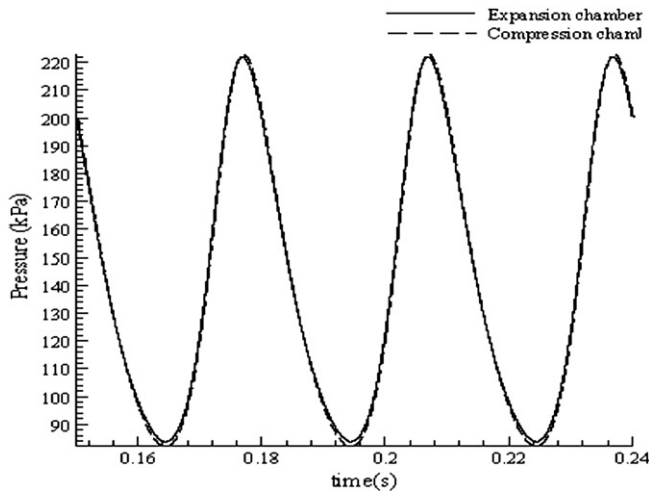


Fig. 7. Pressure variation of the expansion and the compression chambers.

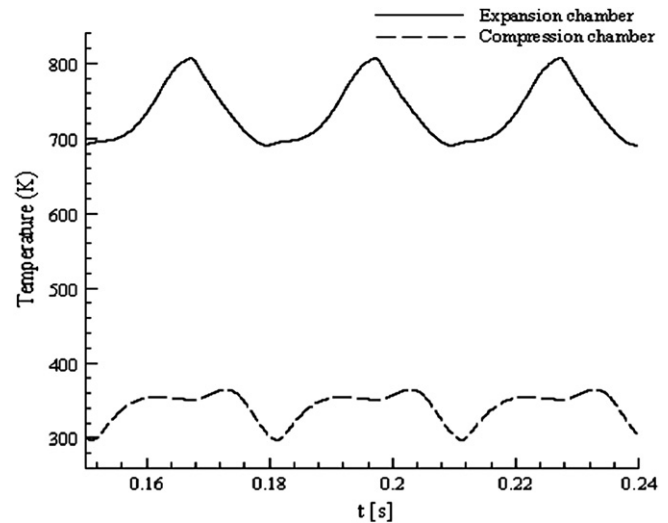


Fig. 9. Temperature variation in the expansion and the compression chambers.

101.3 kPa at $t = 0$. The pressure then is varied periodically during cycles. The pressure difference between these two chambers is one of the driving forces that make the fluid flow through the regenerative channel. For this case, no higher pressure difference is observed since the engine was relatively small in size. The pressure difference data are shown in Fig. 8. Expansion chamber is full of high temperature working gas in a cycle, so the pressure inside is larger than that in the compression chamber usually. However, when the volume of the expansion chamber is further increased, the pressure inside is reduced to a certain low value. In that case, the pressure may be lower in the expansion chamber than in the compression chamber, and the pressure difference becomes negative.

Fig. 9 illustrates the temperature variations in the expansion chamber and compression chamber. The heat source and the heat sink are maintained at 800 K and 300 K, respectively. It is interesting to note that the temperature in the expansion chamber could be higher than 800 K at some instants, and on the contrary, the temperature in the compression chamber could be lower than 300 K. This is because the air in the expansion chamber could be overheated due to compression and the air in the compression

chamber could be overcooled due to expansion of the volume. This model applies the energy equation to the control volumes. By assuming the specific heat of the working gas, air, and the thermal resistance in the expansion chamber and compression chamber, the temperature variation is calculated during cycles. Since the temperatures of heat source and the heat sink are given, the heat transfer in the chambers can be calculated in terms of the thermal resistances and the temperature difference between the fluid temperature and the heat source or heat sink temperature.

Fig. 10 shows the magnitudes of heat transfer rates in the expansion and the compression chambers. In this figure, the value of the heat transfer rate is positive indicating that the heat is transferred from the surroundings to the chamber. On the contrary, the value of the heat transfer rate is negative indicating that the heat is transferred from the chamber to the surroundings. It is observed that the heat transfer rate in the expansion chamber is not always positive and that in compression chamber is not always negative. The reason for this has already been discussed earlier.

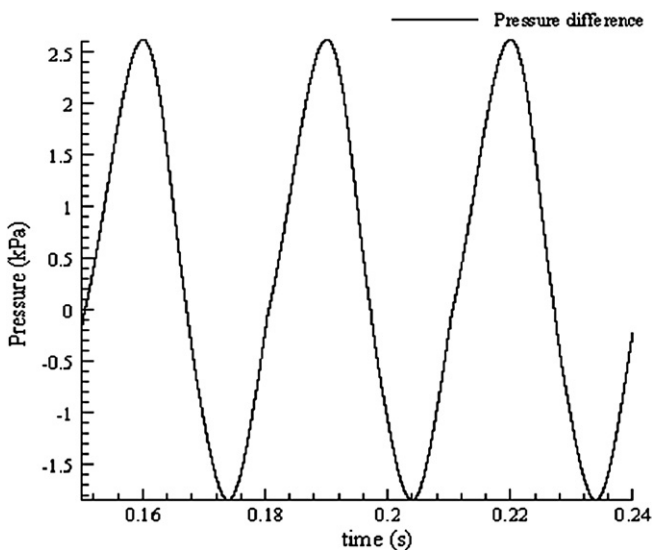


Fig. 8. Pressure difference between the expansion and the compression chambers.

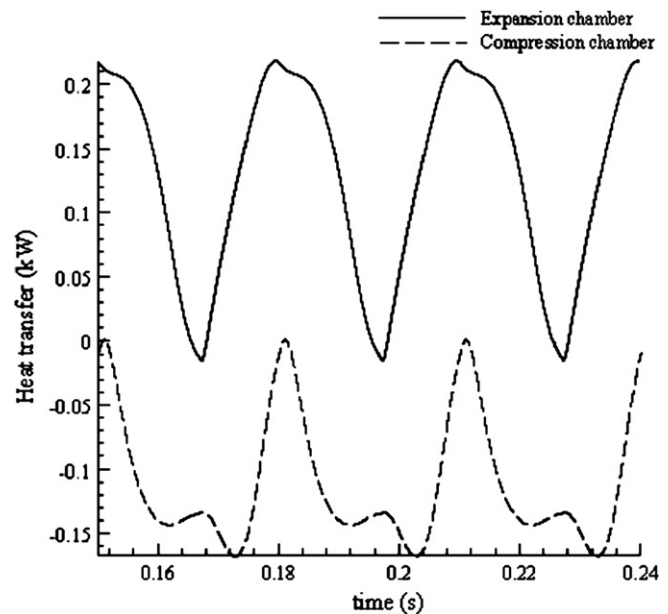


Fig. 10. Heat transfer rate in the expansion and the compression chambers.

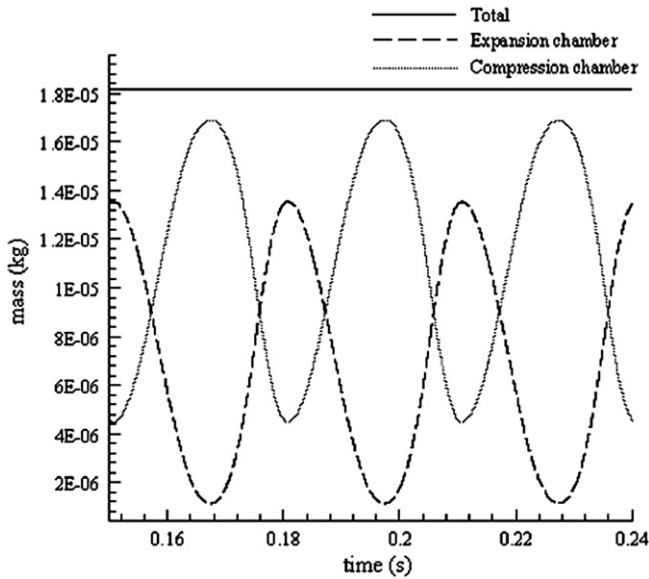


Fig. 11. Mass variation in the expansion and the compression chambers.

Fig. 11 shows the variation in the masses of the working fluid in the expansion and compression chambers. Total amount of mass in the engine remains constant; therefore, as the mass in the expansion chamber is decreased, the mass in the compression chamber will be increased.

Fig. 12 shows the P – V diagrams for the expansion chamber and compression chambers. The areas of the clockwise and counter-clockwise P – V cycles with the expansion and the compression chambers are calculated by integrating $\int_{t_0}^{t_0+t_p} P_e dV_e$ and $\int_{t_0}^{t_0+t_p} P_c dV_c$, respectively. The network output is thus obtained from Eq. (31).

Next, effects of the size of the regenerative channel (G), heat source temperature (T_H), the distance between the two gears (L), and the offset distance from the crank to the center of gear (R_d) are studied and the results are shown in Figs. 13–16.

Fig. 13 shows the effects of the size of the regenerative gap on the power output and thermal efficiency of the engine. The regenerative gap size is varied from 0.00021 to 0.0008 m. The power output and thermal efficiency increase rapidly with the gap size as G is less

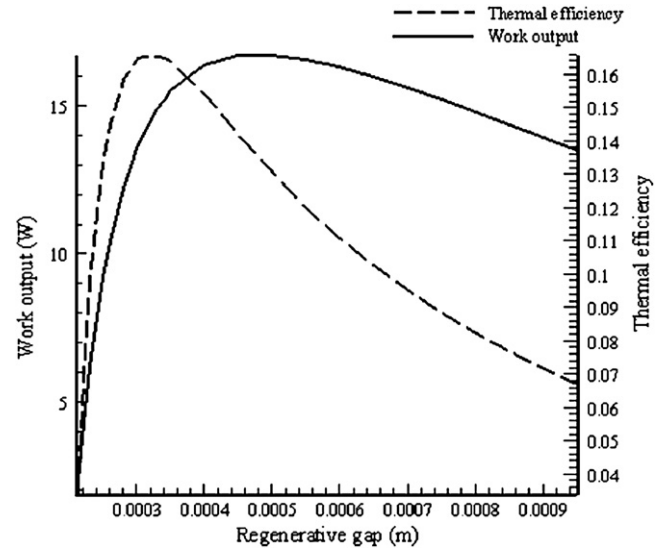


Fig. 13. Effects of the regenerative gap (G).

than 0.0003 m. This is attributed to the significant increase in the shear friction loss as the gap size is reduced. On the other hand, when the gap size is increased, the regenerative effectiveness of the regenerative channels would be reduced. Therefore, there should be an optimal value for the gap size that leads to a peak value of the power output or the thermal efficiency. The curve of the power output indeed reaches a peak value of 16.75 W at $G = 0.0005$ m. The thermal efficiency exhibits the same tendency, and at $G = 0.0003$ m the thermal efficiency reaches its peak value. In this figure, it is found that the maximum value of the thermal efficiency of the engine is 16.5%.

Fig. 14 illustrates the effects of the heat source temperature (T_H) on the engine's power output and thermal efficiency. It is observed that the power output can be increased from 7.96 to 32.78 W as T_H is elevated from 600 to 1200 K. The value of T_H is strictly restricted by the induced thermal stress that the materials can endure. In the figure, the power output seems to be linearly varied with T_H .

The effects of the distance between the center of the gears (L) on the power output and the thermal efficiency of the engine are shown in Fig. 15. It is found that the power output is reduced from

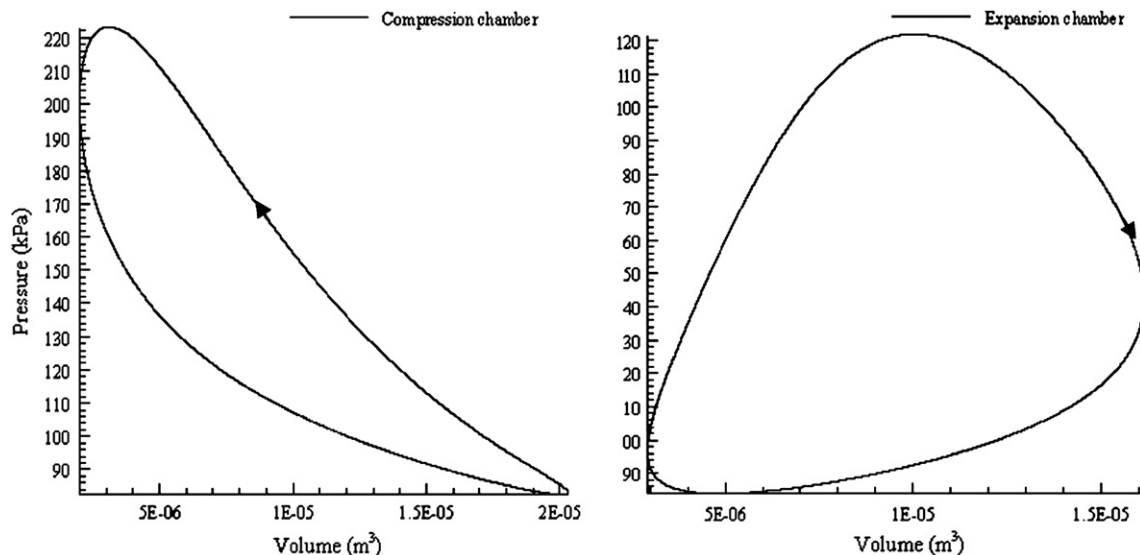


Fig. 12. P – V diagrams for the expansion and the compression chambers.

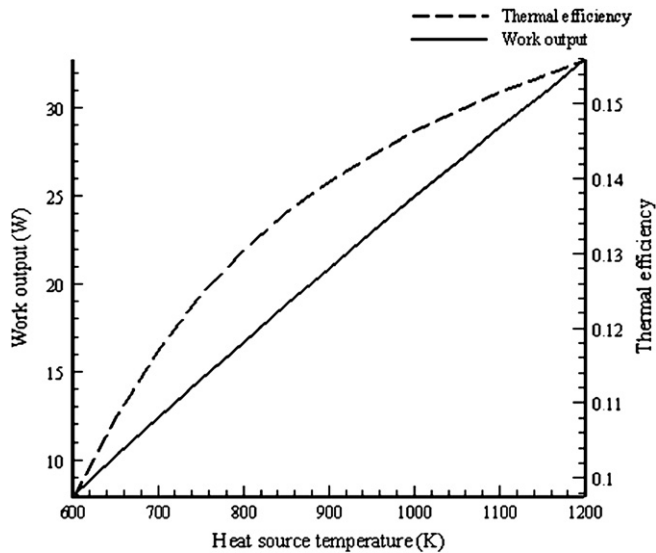


Fig. 14. Effects of heat source temperature (T_H).

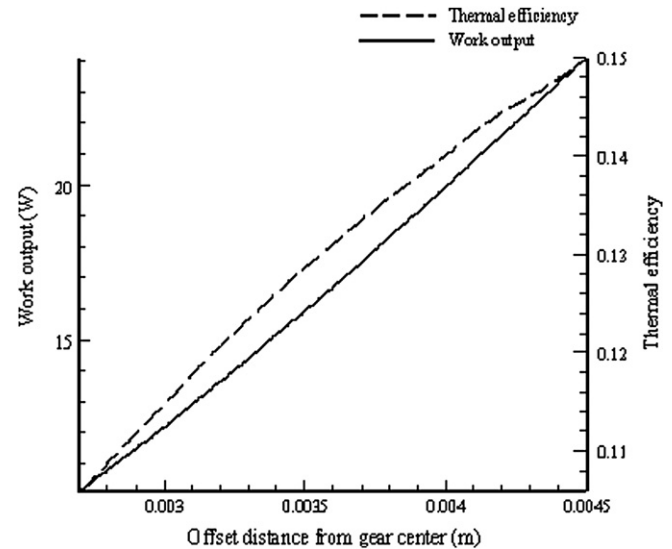


Fig. 16. Effects of the offset distance from the crank to the center of gear (R_d).

18.286 to 15.3 W, and the thermal efficiency from 0.1377 to 0.123, as the value of L is increased from 0.040 to 0.044 m.

Fig. 16 shows the power output and the thermal efficiency as functions the offset distance from the crank to the center of gear (R_d). It is found that as R_d becomes larger, the power output and the thermal efficiency are both increased. Power output can be varied from 10.139 to 24.137 W with R_d changing from 0.0027 to 0.0045 m, while thermal efficiency from 0.106 to 0.150. Obviously, both the power output and the thermal efficiency monotonically increase with R_d .

It is interesting to mention that the distance between the centers of the gears (L) and the offset distance from the crank to the center of gear (R_d) exhibit opposing effects on the performance of engine according to Figs. 15 and 16. These two parameters greatly affect the strokes of the piston and the displacer. In accordance with the obtained results, it is noted that the combination of these two parameters should be carefully designed for different requirements in engine performance.

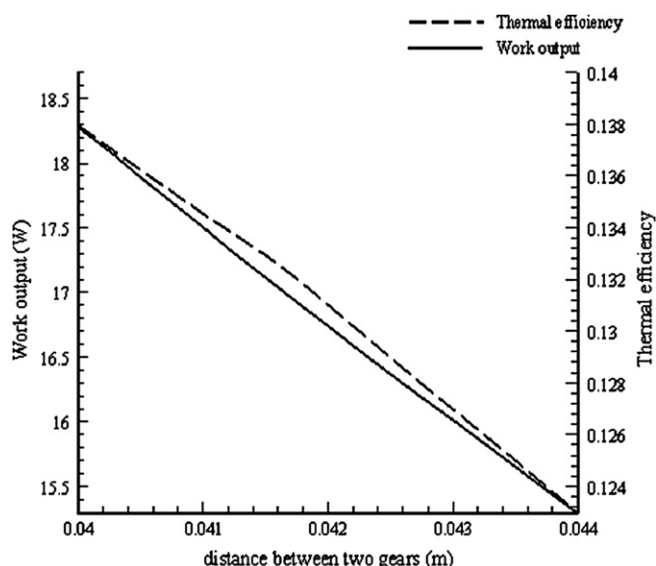


Fig. 15. Effects of the distance between two gears (L).

4. Concluding remarks

A theoretical model which is applied to simulation of thermodynamic cycle and performance of a beta-type Stirling engine with rhombic-drive mechanism has been proposed in this study. Periodic variation of pressures, volumes, temperatures, masses, and heat transfers in the expansion and the compression chambers are predicted, and a parametric study of the dependence of the power output and thermal efficiency on the geometrical and physical parameters for a base-line case is performed.

Results show that by adjusting the influential parameters including regenerative gap, distance between two gears, offset distance from the crank to the center of gear, and the heat source temperature, the performance of the base-line case can be improved. The power output of the base-line case reaches a peak value of 16.75 W at $G = 0.0005$ m, accompanied by a thermal efficiency of only 13.1%. If the thermal efficiency is of major concern, the thermal efficiency can be elevated to a peak value of 16.5% at $G = 0.0003$ m.

It is also observed that the power output of the base-line case can be increased from 7.96 to 32.78 W as T_H is elevated from 600 to 1200 K. However, the value of T_H is strictly restricted by the induced thermal stress that the materials can endure.

Meanwhile, an increase in the distance between the center of the gears (L) leads to a decrease in the power output and the thermal efficiency. On the other hand, it is found that the power output and the thermal efficiency both increase with the offset distance from the crank to the center of gear (R_d). The two parameters, L and R_d , exhibit opposing effects on the performance of engine.

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